

Comment on “Capturing correlations in chaotic diffusion by approximation methods”

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In their paper [1], Knight and Klages argue that the application to their one-dimensional map model of the approximation scheme for the characterization of transport coefficients of diffusion processes developed by the present authors in Refs. [2] is hampered by the presence of states with zero velocity. This observation would seem like a serious limitation of our formalism, but, as we illustrate below, it is in fact inaccurate, and is due rather to a misinterpretation of the Machta-Zwanzig approximation [3], based on a confusion between two distinct timescales.

Indeed, consider a simple symmetric random walk on a one-dimensional lattice with unit spacing $\ell = 1$ between neighboring sites, in which a walker hops at every unit time step left or right with probability p , $0 < p < 1/2$, and stays put with probability $q = 1 - 2p$. It is a trivial calculation to show that the mean squared displacement per unit step of such a walker is $2p$, so that the diffusion coefficient of this process is simply $D = p$.

The same result is equally obtainable by application of the Machta-Zwanzig formula [3]. This dimensional prediction, according to which (in one spatial dimension) the diffusion coefficient is one half of the square of the length ℓ of the walker’s displacements divided by their timescale τ , must indeed be exact for this simple model, since walkers have no memory of their history. Accordingly, the

diffusion coefficient is the lattice spacing squared (which we took to be unity), divided by twice the timescale of jumps. That timescale is the average time spent by a walker at any given site before it moves on to a neighboring one. In this case, it is easily computed to be $\tau = 1/(2p) > 1$. This recovers the previously-stated result.

In other words, the diffusion coefficient of the symmetric random walk described above is equal to that of another, yet simpler, process, in which a walker jumps left or right with probability $1/2$ at rate $2p$ (whether in continuous or discrete time). Ignoring the underlying dynamics, these quantities are easily accessible to direct measurements, which is why this prediction is so useful.

The flaw in the attempt of Knight and Klages [1] to transpose our formalism [2] to their model is to have confused the timescale of displacements with the unit timescale of the underlying process. Zero-velocity states are in fact irrelevant, since they are accounted for by plugging in the relevant timescale, that of displacements. Our formalism is but a simple extension of the above considerations, which allows for the systematic inclusion of memory effects in such processes in a self-contained way. It is in fact very general and is easily applied by direct measurement of the appropriate quantities.

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